Advanced Ecology of Isle Royale Lesson Plans

Trapezoids, Area, and Moose Population Estimates

High School Math with a writing component

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Introduction

Moose population estimates on Isle Royale are made by counting moose in 91 specific areas and then using those counts to establish the estimated population on the entire island. The counts are adjusted based on a study of radio collared moose (what percent of radio collared moose were actually spotted by the spotter when the pilot knew they were in the plot). My interest lies in the area portion of the estimate. The researchers are using a topographical map to establish the counting plots, but are using the two dimensional area of the map for their moose estimate. My goal with these lessons, is to compare the two dimensional area of the mapped plot to an estimate of the actual surface area of the plot using the topography on the map and trapezoidal rule concept of estimating area in calculus.

The project will take four days of class and will be broken down according to the schedule:

Day 1:

- Discuss Trapezoidal Rule for estimating area under a curve without a known, or easily integrable, function.

Day 2:

- Discuss Isle Royale population count procedure (general)
- Discuss measuring the bases of the trapezoid on a topographical map using Pythagorean Theorem taking into account topography.

Day 3:

- Using concepts from yesterday to measure the area of a mapped plot

Day 4:

- Discuss errors in measurements and procedure.
- Would these measurements represent minimum or maximum areas? (theoretically)

These lessons will address the Michigan Department of Education (2006) Benchmarks:

- E1.1B *Evaluate the uncertainties or validity of scientific conclusions using an understanding of sources of measurement error…*
- G1.2.3 *…use the Pythagorean Theorem … to solve mult-step problems*
- G1.4.1 *Solve multistep problems … involving…area of … trapezoids.*

Materials

The materials needed for these lessons include textbook with practice problems of the trapezoidal rule (Stewart, 2003; or Larson & Hostetler & Edwards, 1998; or Larson & Hostetler & Edwards, 2002*;* would be suitable), topographical map of Isle Royale with the 91 population sample areas drawn in, measuring device, (I am going to have students experiment here to see what they can come up with for the most precision and accuracy see details of the lesson for ideas.) calculator, and possibly a computer with Excel for repetitive number crunching.

Daily Lesson Plans

Day 1

Objective

Students will be able to use the Trapezoidal Rule to find the area between two functions.

Procedure

Review the idea of integration being the area between the curve and the x-axis. Then discuss finding the area of the curve when the function is not easily integrable. What options would there be. Of course one could use the limit of a Riemann Sum as the norm approaches zero, which is the definition of a definite integral (Larson & Hostetler & Edwards, 1998; Larson & Hostetler & Edwards, 2002). However, that is a lengthy and somewhat cumbersome process on more complicated functions. There are easier ways. One of which is to use trapezoids to estimate the area under the curve rather than rectangles which the Riemann Sum uses. The trapezoidal rule is easily developed using any simple function and sub-divisions of equal width. The students have a problem sometimes seeing that dividing a function vertically and connecting the end of the divisions results in trapezoids until you tell them that the bases of the trapezoids are the vertical lines. The area of a single trapezoid can be found by using the formula $A = \frac{b_1 + b_2}{2} h$ where b_1 and b_2 represent the bases of the trapezoid (in this case the height of the function at the point or $f(x)$) and h represents the height of the trapezoid perpendicular to the bases (i.e. the distance between the vertical lines).

In the case where the distance between the divisions of the function are equal, where h is the same for all trapezoids, these trapezoid areas can be added as follows where n represents the number of the last division.

$$A = \frac{b_1 + b_2}{2} h + \frac{b_2 + b_3}{2} h + \frac{b_3 + b_4}{2} h + ... + \frac{b_{n-2} + b_{n-1}}{2} h + \frac{b_{n-1} + b_n}{2} h$$

This will simplify to

$$A = \frac{h}{2}\left(b_1 + b_2 + b_2 + b_3 + b_3 + b_4 + ... + b_{n-2} + b_{n-1} + b_{n-1} + b_n\right).$$

As can be seen in the above equation, the right base of one trapezoid is the left base of the next trapezoid and the formula can be adjusted to

$$A = \frac{h}{2}\left(b_1 + 2b_2 + 2b_3 + 2b_4 + ... + 2b_{n-1} + b_n\right)$$

This concept can now be used to estimate the area of any function when we know the height of the function at various points and the distance between those various points. At this point I usually assign problems from the student text book *Calculus of a Single Variable* (Larson & Hostetler & Edwards, 1998). However, the text book is not necessary. Problems can be made up easily by drawing a field, pond, or whatever and giving the students the distances across at various points equidistant apart. The "trick" is to remember that if you start 20 feet from an edge and the measurements across are taken every 20 feet, the first and last measurement are usually 0. Sometimes students forget the first trapezoid with one base 0 (which is a triangle!). See Appendix A for a sample problem and its solution.

Day 2

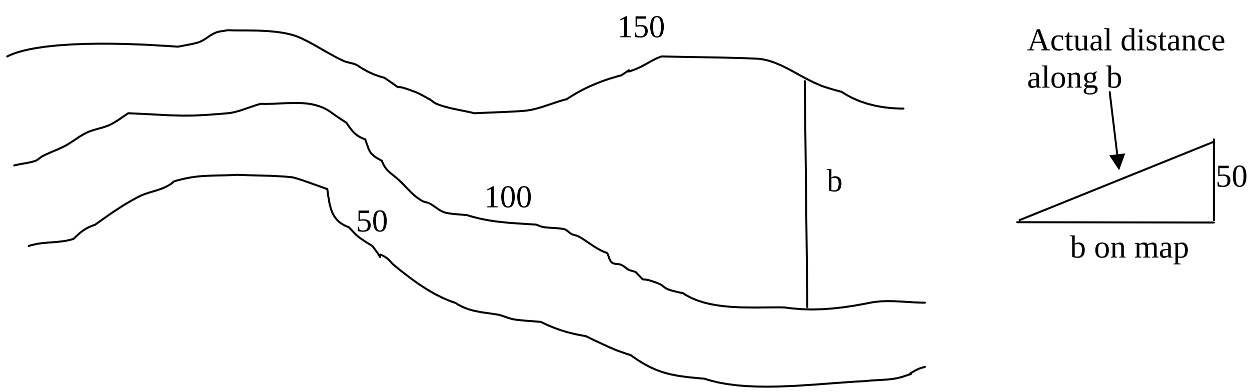
Objectives

- Students will have an understanding how area is used to estimate moose population on Isle Royale.
- Students will be able to calculate the length of the bases of trapezoids on a topographical map using Pythagorean Theorem to take into account topography.

Procedure

The class will begin with a discussion of the moose population on Isle Royale and how that population is estimated each year. Of special interest will be the area of the 91 plots on Isle Royale and how those areas are used in the calculation of the population. Students will then be lead in a discussion of the island being three dimensional, but only using two dimensions for measuring the areas on the maps. That is, a box is drawn on the map, and then the area of the box is measured using the map key. The point of this discussion is to demonstrate that there might be error in the population estimates, because there is error in the area measurements.

The next discussion would involve how we could use the trapezoidal rule in finding the area of the plots taking into account the topography. Parallel lines would have to be drawn across the plots, and then we would have to measure the length of the lines between the topographic lines, then these measurements would have to be converted to the actual lengths using the Pythagorean Theorem. The map diagram might be of help to describe how the Pythagorean Theorem would apply.



Let's assume that the line b is 80 feet long according to the map key. How long is b taking into account topography? That is, it is 80 feet along the path of b as the crow flies, but what is the measure of the hypotenuse of a right triangle with one side of 80 (the length of b measured on the map) and a height of 50 (the difference in elevation from one line to the next). b according to the topography would be calculated as

$$b = \sqrt{80^2 + 50^2} = \sqrt{6400 + 2500} = \sqrt{8900} \approx 94 \text{ feet.}$$

I would now give the students practice problems using different lengths of b and also different values of topography as some maps may have the topography in 10 ft intervals, others in 1000 ft intervals, or any values in between. See Appendix A for a sample problem and solution.

Day 3

Objective

- Students will be able to use the concepts developed on days 1 and 2 to estimate the area of a plot on a topographical map.

Procedure

Give each group of two to four students a plot to find the area of using the procedure developed over the last two days. The plot could consist of a topographical map with the area marked, a scanned copy, or high resolution digital photo of the map with the area marked. If a scanned map or digital photo is used, make sure that the map key is at the same resolution as the map itself. I believe that a scanned map would work better as students would be able to zoom in on the scanned area and even count pixels if necessary to estimate the distances on the map and convert the map lengths to actual dimensions. The students may wish to program their calculators, or use a formula in an Excel spreadsheet to convert the measurements.

Day 4

Objectives

- Students would be able to describe whether the estimates they made would be theoretically maximums or minimums for the given areas (assuming no measurement error).
- Students will be able to discuss errors in measurements and procedures used in their estimating the area of the plots used for the estimation of the moose population on Isle Royale.

Procedure

Today, students would lead a discussion on what their calculations of area would represent in the real world. Focus would be on whether their work would represent the minimum or maximum area, and why. (The answer is minimum as the trapezoid would represent straight line distances between points which is the shortest distance.) They would also be asked to discuss errors in their measurements and how the activity could be done in the future to minimize those errors.

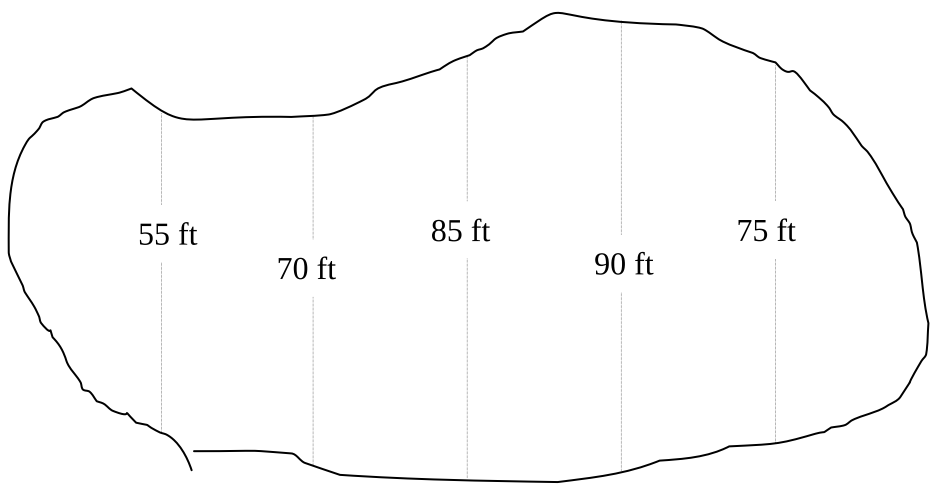
Unit Assessment

The assessment piece of these lessons will be a graded persuasive paper. The paper will be written toward the researchers on Isle Royale and should try to persuade them to adjust their population counting method to include topography in the measurement of plot area or against it. The students will be required to describe how they calculated the area of their plot taking into account the topography. Students will be asked how they think that their measurements might affect the population estimates made by the researchers on Isle Royale. They will also be asked to describe the possible errors in their measurements or calculations and what could be done in the future to minimize them. See Appendix B for this formal assignment.

Appendices

Appendix A: Sample Problems with solutions

- Day 1 Problem
  - Estimate the area of a pond given the following measurements taken every 25 feet across the pond.



  - Solution: Since the first and last measurements are 0 we need to double each of the measurements given, add them together, and then multiply by 25/2 or 12.5. We would have 12.5(110 + 140 + 170 + 180 + 150) = 12.5(750) = 25(375) = 9375 square feet.

- Day 2 Problems
    - Use the Pythagorean Theorem to fill in the missing values on the table below.

      (Round your answers to the nearest whole number.)

Problem	Distance measured on map	Difference in Elevation	Distance on ground
1	30 ft	50 ft	58 square feet
2	12 m	20 m	23 square meters
3	3.2 ft	25 ft	25 square feet
4	12 ft	10 ft	16 square feet
5	150 m	10 m	150 square meters
6	1453 m	100 m	1456 square meters
7	2430 ft	100 ft	2432 square feet
8	1875 ft	50 ft	1876 square feet
9	2450 m	50 m	2451 square meters
10	8738 m	25 m	8738 square meters

Appendix B – Unit Assessment

Persuasive Essay

You must write a persuasive essay toward the researchers on Isle Royale. Your essay should try to persuade them toward or against adjusting their population counting method to include topography in the measurement of the plot areas. You are required to describe how you calculated the area of the plot taking into account the topography. You should discuss why you think that your measurements might or might not affect the population estimates made by the researchers on Isle Royale. You should also describe the possible errors in your measurements or calculations and what could be done in the future to minimize them.

Bibliography

Larson, R.E., Hostetler, R. P., & Edwards, B. H. (1998). Calculus of a Single Variable (6th ed.) Boston, MA: Houghton Mifflin

Larson, R.E., Hostetler, R. P., & Edwards, B. H. (2002). Calculus of a Single Variable (7th ed.) Boston, MA: Houghton Mifflin

Michigan Department of Education. (October, 2006). High School Content Expectations: Science: Earth Science. Retrieved August 30, 2007 from http://www.michigan.gov/documents/Earth_HSCE_168206_7.pdf

Michigan Department of Education (August, 2006). Michigan Merit Curriculum: Course/Credit Requirements: Geometry, Retrieved August 30, 2007 from http://www.michigan.gov/documents/Geometry_167749_7.pdf

Stewart, J. (2003).Calculus (5th ed.). Belmont, CA: Brooks/Cole – Thomson Learning